Adding modalities to many-valued logics

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Many-valued modal logics combine the Kripke semantics of classical modal logics with a many-valued semantics at each world, in order to model epistemic, spatio-temporal, and other modalities in the presence of vagueness or uncertainty.

Two core families of many-valued modal logics emerge: *"order-based" modal logics*, including modal extensions of Gödel logics [3, 8, 4, 2], where only the order type of the truth values matters, and *"continuous" modal logics*, such as those based on Łukasiewicz logic [6, 1, 7], where propositional connectives are interpreted by continuous functions over sets of real numbers.

Although many-valued modal logics are easy to define semantically – simply decide on a suitable set of values and operations – they are not so easy to study. In this talk we focus on the axiomatization of these logics.

There are challenging problems for both order-based modal logics and continuous modal logics. For example, an axiomatization for the Gödel modal logic over many-valued frames is provided in [4], but as yet no axiomatization is known for the Gödel modal logic over standard (Boolean-valued) frames. Finite-valued Łukasiewicz modal logics are axiomatized in [7], but the axiom system provided for the infinite-valued Łukasiewicz modal logic includes a rule with infinitely many premises.

As an intermediary step towards finding a finitary axiomatization for infinite-valued Łukasiewicz modal logic, we present a simple prototype many-valued modal logic of magnitude $K(\mathbb{R})$ that extends the multiplicative fragment of abelian logic [5]. We provide a sound and complete axiom system for $K(\mathbb{R})$, making use of both a labelled tableaux system and a sequent calculus admitting cut elimination to establish the more difficult completeness result. The next step would then be to interpret infinite-valued Łukasiewicz modal logic in a suitable extension of $K(\mathbb{R})$ with lattice connectives.

References

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