Implicative-groups vs. groups and generalizations

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We present the implicative-group and the partially-ordered (lattice-ordered) implicative-group
as a term equivalent definition of the group and the partially-ordered (lattice-ordered) group,
respectively. The lattice-ordered implicative-group is the great piece which missed from the
puzzle showing the connections between lattice-ordered groups and some algebras of logic. We
establish connections at group level and connections between the group level and the algebras of
logic level. We discuss about the filters/ideals and the deductive systems of the involved notions.
We study the normal filters/ideals and the compatible deductive systems, the representability of
some of the involved algebras and we establish other connections between the group level
and the algebras of logic level. Finally, we introduce and study the implicative-states and the
Bosbach-states on l-groups with strong unit.

We introduce for the first time the (sharp) hoops, as generalizations of the groups, and the
(strong) implicative-hoops, as generalizations of the implicative-groups, and we show their term
equivalence.

We introduce for the first time the pseudo-M algebras as generalizations of pseudo-BCI and
pseudo-BCK algebras and we make the connections with the implicative-groups and the
implicative-hoops.

We introduce for the first time the quasi-pseudo-M algebras as generalizations of pseudo-M
algebras. We introduce for the first time the (involutive) quasi-implicative-groups and the (strong
involutive) (super) quasi-implicative-hoops, as generalizations of implicative-groups and
implicative-hoops, respectively.

We introduce for the first time the quasi-m algebras, in particular the quasi-m unital magmas, the
quasi-m monoids, the (involutive) quasi-m groups and the (strong involutive) (super) quasi-m
hoops, as generalizations of unital magmas, monoids, groups, hoops, respectively.
Connections are then established with the quasi-algebras.

The whole picture is completed by the examples and the informations provided by over 100
programs in PASCAL programming language.

We believe that the new notions and the new connections, presented in an unifying way in the
monograph, open new perspectives in the study of unital magmas, on the one hand, and of
algebras of logic (of pseudo-M algebras, in particular), on the other hand.