Mathematical foundations for conceptual blending

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Conceptual blending

This work constitutes an effort to provide adequate mathematical foundations to *conceptual blending*, which is an important research problem in the area of *computational creativity*. This is a relatively recent multidisciplinary science, with contributions from/to artificial intelligence, cognitive sciences, philosophy and arts, going back at least until to the notion of *bisociation*, presented by Arthur Koestler. Its aims are not only to construct a program that is capable of human-level creativity, but also to achieve a better understanding and to provide better support for it. Conceptual blending was proposed by Fauconnier and Turner as a fundamental cognitive operation of language and common-sense, modelled as a process by which humans subconsciously combine particular elements of two possibly conceptually distinct notions, as well as their relations, into a unified concept in which new elements and relations emerge.

The structural aspects of this cognitive theory have been given rigorous mathematical grounds by Goguen based upon category theory. In this formal model, concepts are represented as logical theories giving their axiomatization. Goguen used the algebraic specification language OBJ to axiomatize the concepts, a language that is based upon a refined version of equational logic; but in fact the approach is independent of the logical formalism used (this is why category theory is involved).

In the above-mentioned work by Goguen there are convincing arguments, supported by examples, for the partiality of theory translations, which represents very much a departure to a different mathematical realm than that of logical theories (even when considered in a very general sense, as commonly done in modern computer science). In category-theoretic terms, this means that we need to consider there categories equipped with partial orders on the hom-sets that are preserved by the compositions of arrows/morphisms. These are special instances of 2-categories (a rather notorious concept), somehow half-way between ordinary categories and 2-categories; according to Goguen, this is what motivates the term

$\frac{3}{2}$-*category*. To summarise the main mathematical idea underlying theory blending as it stands now:

> *Theory blending is a cocone in a $\frac{3}{2}$-category in which objects represent logical theories and arrows correspond to partial mappings between logical theories.*

There is still a great deal of thinking on whether the cocone should actually be a colimit (in other words, a minimal cocone) or not necessarily. An understanding of this issue is that blending should not necessarily be thought as a colimit, but that colimits are related to a kind of *optimality* principle. Moreover, since $\frac{3}{2}$-category theory has several different concepts of colimits, there is still thinking about which of those is most appropriate for modelling the blending operation.

Goguen's ideas about theory blending benefited from an important boost with the European FP7 project COINVENT that has adopted them as its foundations. Based on this, a creative computational system has been implemented and demonstrated in fields like mathematics and music (although both use the strict rather than the $\frac{3}{2}$-version of category theory).

$\frac{3}{2}$-institutions

However, the COINVENT approach still lacks crucial theoretical features, especially a proper semantic dimension. Such a dimension is absolutely necessary when talking about concepts because meaning and interpretation are central to the idea of concept. For example, the idea of consistency of a concept depends on the semantics. If one considers also the abstraction level of Goguen's approach in its general form, of non-commitment to particular logical systems, then *the institution-theoretic dimension appears as inevitable*. In fact, Goguen argued for the role of institution theory in and so does the COINVENT project. However, institution theory cannot be used as such in a proper way because, as it stands now, it cannot capture the partiality of theory morphisms (which boils down to the partiality of signature morphisms).

Therefore we define a $\frac{3}{2}$-categorical extension of the concept of institution, called $\frac{3}{2}$-*institution*, that accommodates those aspects and that starts from an abstract $\frac{3}{2}$-category of signatures. Moreover, based on this, we unfold a theory of $\frac{3}{2}$-institutions aimed as a general institution theoretic foundations for conceptual blending.